

# Renormalization Group and Black Hole Production in Large Extra Dimensions

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It has been suggested that the existence of a non-Gaussian fixed point in general relativity might cure the ultraviolet problems of this theory. Such a fixed point is connected to an effective running of the gravitational coupling. We calculate the effect of the running gravitational coupling on the black hole production cross section in models with large extra dimensions.

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The overwhelming success of the standard model of particle physics in providing a consistent description of strong and electroweak interactions encourages further steps towards the unification of all forces. A next step could be the unified description of the standard model and the general relativity, which can be derived from the Einstein-Hilbert action. In order to unify the two theories one needs to solve two major problems.

One problem is encountered, when the assumption is made that general relativity and the standard model have their origins in a single unified field theory X with a single unified mass scale  $M_X$ . It is not understood why the mass scales of general relativity ( $m_{Pl}$ ) and of the standard model ( $m_H$ ) are so different in nature. In fact, a simple estimate shows that

$$\frac{m_H}{m_{Pl}} \sim 10^{-17} \quad . \quad (1)$$

This difference is known as the hierarchy problem. Since the gravitational coupling is  $G_N \sim 1/M_{Pl}^2$ , this leads to the question: "Why is gravity so weak as compared to the other forces in nature?". The hierarchy problem, which is shown in Eq. (1), can be resolved by either a Higgs mass of the order of  $10^{19}$  GeV rather than the expected few hundred GeV [1] or by lowering the Planck mass down to the TeV region. However, a higher Higgs mass would aggravate the hierarchies between  $m_H$  and the light fermions. This attempt would then create a new hierarchy by eliminating the other. Lowering the Planck scale, however, would be much more desirable. In the context of extra dimensions there exist scenarios that give rise to a lower Planck scale and explain the difference in Eq. (1). Although, an explanation for the mass hierarchy does not imply that the unified theory X has been found, it might give a useful hint on how to proceed. In reference [2, 3, 4] Arkani-Hamed, Dimopoulos, and Dvali do this by assuming that the additional spatial dimensions are

compactified on a small radius  $R$  and further demanding that all known particles live on a  $3+1$  dimensional sub-manifold (3-brane). They find that the fundamental mass  $M_f$  and the Planck mass  $m_{Pl}$  are related by

$$m_{Pl}^2 = M_f^{d+2} R^d \quad . \quad (2)$$

Within this approach it is possible to have a fundamental gravitational scale of  $M_f \sim 1$  TeV. The huge hierarchy between  $m_H$  and  $m_{Pl}$  would then come as a result of our ignorance regarding extra spatial dimensions.

Another problem is the bad ultraviolet behavior of gravity. The standard approach to the quantization of general relativity with the metric  $g_{MN}(x)$  includes perturbations  $h_{MN}(x)$  (gravitons) around the flat Minkowski metric  $\eta_{MN}$  as the local quantum degrees of freedom [5]

$$g_{MN}(x) = \eta_{MN} + h_{MN}(x) \quad . \quad (3)$$

The standard loop expansion in the gravitational coupling, fails because every new order in the perturbative expansion brings new ultraviolet (UV) divergent contributions to any physical process. Since the Einstein-Hilbert action contains operators of mass dimension higher than four, those divergences cannot be cured by the standard renormalization procedure used in the standard model. A solution to this problem would be found if one could show that the poor UV behavior is not present in the full theory but only comes about due to the expansion in the gravitational coupling. According to this point of view, the full quantum gravity would have an energy dependent asymptotically safe coupling constant with a non-Gaussian fixed point. Such renormalization group (RG) techniques have been successfully used for a number of different problems [6, 7, 8, 9, 10].

A combination of these two approaches would solve both, the hierarchy problem and the UV problem. Since the existence of a non-Gaussian fixed point in higher dimensional gravity was shown in [11, 12], an implementation of the running coupling into theories with large extra dimensions [13, 14] was possible. Moreover, RG effects on graviton production, graviton exchange, and Drell-Yan processes in the context of extra dimensions have been considered in [15, 16].

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In this paper we study how RG affects the possible production of microscopical black holes (BH), which is probably the most prominent collider signal for large extra dimensions.

A complete understanding of all BH properties is only possible in a unified theory of quantum-gravity. In the framework of large extra dimensions the metric of a black hole with mass  $M$  is given by

$$ds^2 = -\sqrt{1 - \frac{16\pi M}{(d+2)A_{d+2}m_{Pl}^2}r^{d+1}}dt^2 + \frac{1}{\sqrt{1 - \frac{16\pi M}{(d+2)A_{d+2}m_{Pl}^2}r^{d+1}}}dr^2 + r^2 d\Omega_{d+2}^2, \quad (4)$$

where  $A_{d+2}$  is the  $d+2$  dimensional sphere

$$A_{d+2} = \frac{2\pi^{\frac{3+d}{2}}}{\Gamma(\frac{3+d}{2})}. \quad (5)$$

Due to the low fundamental scale  $M_f \sim \text{TeV}$  and the hoop conjecture [17], it might be possible to produce such objects with mass of approximately 1 TeV in future colliders [18, 19, 20, 21]. This can only be the case when the invariant scattering energy  $\sqrt{s}$  reaches the relevant energy scale  $M_f$ . The higher dimensional Schwarzschild radius [20, 22] of these black holes is given by

$$R_H^{d+1} = \frac{16\pi(2\pi)^d}{(d+2)A_{d+2}} \left(\frac{1}{M_f}\right)^{d+1} \frac{M}{M_f}. \quad (6)$$

This would open up a unique possibility of studying quantum gravity in the laboratory. A semi-classical approximation for the BH production cross section is given by

$$\sigma(M) \approx \pi R_H^2 \theta(\sqrt{s} - M_f), \quad (7)$$

where the theta function ensures that black holes are only produced above the  $M_f$  threshold. This threshold condition is necessary because a black hole with  $M < M_f$  would not be well defined, as it would have for example a temperature  $T > M$ . Such a cut will have crucial significance in the RG approach to BHs. The validity of this approximation has been debated in [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. Still, improved calculations including the diffuseness of the scattering particles (as opposed to point particles) and the angular momentum of the collision (as opposed to head on collisions) as well as string inspired arguments only lead to modifications of the order of one [34, 35, 36, 37]. However, there are arguments that the formation of an event horizon can never be observed [38, 39].

Non-perturbative renormalization is performed in Euclidian spacetime and has been successfully applied to a variety of field theories such as quantum chromodynamics [40] and gravity [7, 41]. In the case of gravity, it offers

a possible solution for the problem of non renormalizable UV divergences in the perturbative approach. This solution appears due to the possible existence of a Gaussian fixed point in the UV regime and a non-Gaussian fixed point in the infrared regime.

The main idea in this approach is that it is possible to introduce an infrared cutoff operator in the theory, which leaves the effective Lagrangian invariant under general diffeomorphism transformations. After a gauge fixing it was possible to derive an exact evolution equation for the effective action [6]. Recently this method has been generalized to more than three spatial dimensions [11, 12] and applied to models with large extra dimensions [15, 16]. Both approaches show that cross sections in extra dimensional theories, which originally had a non-unitary behavior for  $\sqrt{s} \gg M_f$ , are now well defined in this high energy limit.

In [10, 15] the running gravitational coupling  $\tilde{M}_f(\sqrt{s})$  is

$$\tilde{M}_f(\sqrt{s}) = M_f \left(1 + \left(\frac{s}{t^2 M_f^2}\right)^{\frac{d+2}{2}}\right)^{\frac{1}{d+2}}. \quad (8)$$

It turns out that  $t$ , which can be obtained from a series of non-trivial integrals, is of the order one in the relevant region of parameter space [15]. This running coupling has two asymptotic regimes. For low energies  $M_f \gg \sqrt{s} \gg 1/R$  one obtains  $\tilde{M}_f(\sqrt{s} \rightarrow 1/R) = M_f$ , whereas, for very high energies  $\sqrt{s} \gg M_f$  one sees that the effective higher dimensional Planck mass diverges  $\tilde{M}_f(\sqrt{s} \gg M_f) \approx \sqrt{s}/y \approx \sqrt{s}$ . This diverging mass scale corresponds to a vanishing gravitational coupling, which is exactly the desired asymptotic safety.

For three spatial dimensions the RG effects on the decay of astronomical black holes have already been discussed in [42]. Surprisingly enough, it turned out that RG effects slow down the Hawking evaporation until a stable black hole remnant is formed. A prediction, which was also made using different arguments for the extra dimensional BH's at the large hadron collider [43, 44, 45, 46]. However, before considering the decay of mini black holes, the RG effects on the formation of black holes should be studied.

This can be done relatively straight forward, by plugging Eq. (8) into Eq. (7). One finds the cross section for the case of a running gravitational coupling

$$\tilde{\sigma}(\sqrt{s}) \approx \frac{\pi}{\tilde{M}_f^2(\sqrt{s})} \left(\frac{16\pi(2\pi)^d \sqrt{s}}{(d+2)A_{d+2}\tilde{M}_f(\sqrt{s})}\right)^{2/(d+1)} \theta(\sqrt{s} - \tilde{M}_f^2(\sqrt{s})). \quad (9)$$

The running Planck scale in Eq. (9) has three consequences. First, for  $\sqrt{s} \sim M_f$  the higher dimensional Planck mass is enhanced by a factor of  $(1 + t^{-d-2})^{1/(d+2)}$  which lowers the area only by a factor of  $(1 + t^{-d-2})^{-2(d+2)^2/(d+2)}$ . As long as  $t$  is of order one, this

corresponds to a change of just a few percent. Secondly, for very high energies  $\sqrt{s} \gg M_f$ , the asymptotic safety wins over the increased energy and the BH area goes to zero  $\sigma \sim 1/s$   $(\sqrt{s}/M_f)^{-1/(d+1)}$ . In Fig. (1) the BH area is shown as a function of  $\sqrt{s}$  for the cases with and without RG effects. The area drops off when  $\sqrt{s}$  is large and, therefore, looks like a “black hole resonance”. The

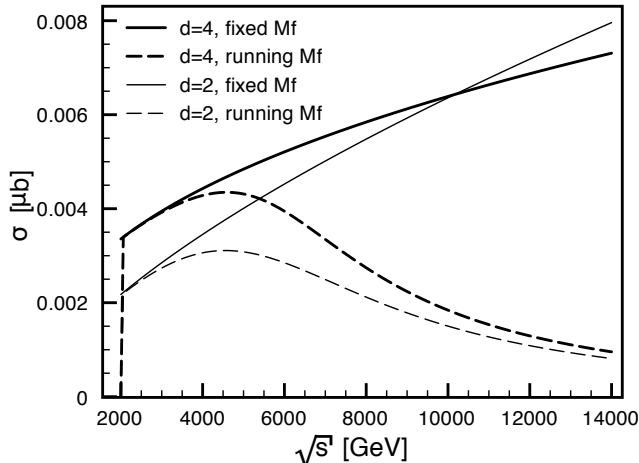


FIG. 1: BH area in  $\mu b$  for  $M_f = 2000$  GeV and  $t = 3$  as a function of  $\sqrt{s}$ .

third consequence of the running  $\tilde{M}_f$  is on the threshold condition. The theta function in (9) gives

$$\sqrt{s} = \left( \frac{t^2}{t^2 - 1} \right)^{1/(d+2)} M_f, \quad (10)$$

which shows that there are dramatic consequences for the standard picture of BH production as soon as  $t$  is of order one. For large  $t \gg 1$  the threshold  $C$  will basically be just slightly raised above  $M_f$  but as soon as  $t$  approaches one the shift increases until when  $t \leq 1$  black holes no longer produced. The sharp behavior of the threshold  $C$  as a function of  $t$  is shown in Fig. (2) for various  $d$ . This cutoff behavior means that the cross section (9) approaches the standard cross section (7) only in the limit where  $t \rightarrow \infty$  and is zero for  $t \leq 1$ , as shown in Fig. 3.

We applied RG techniques to the black hole production scenario in the context of large extra dimensions. We found two surprising effects. First, the area of the black hole, which is of the same order of magnitude as the production cross section, is not only UV safe (as it was observed in standard scattering cross section) but it is damped so strongly that it goes to zero. Secondly, the truncation parameter  $t$ , which does not play an important role in the qualitative standard scattering cross section picture, is very important for the BH threshold. Moreover, BH production could be completely forbidden for  $t \leq 1$ , which according to [15] is perfectly possible.

In the simplest picture of BH production the RG has dramatic consequences. Further study is needed to check

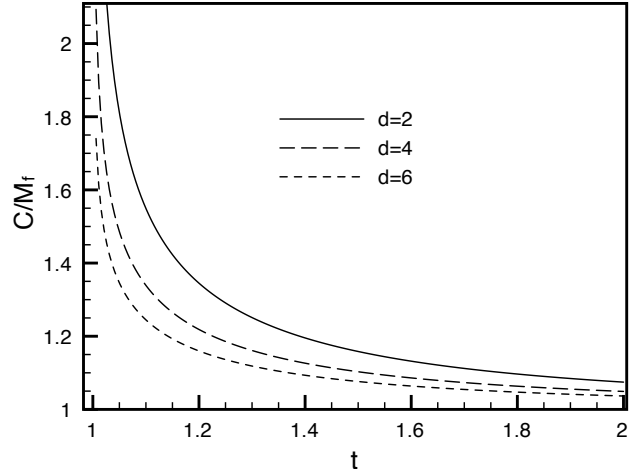


FIG. 2: Threshold  $C$  normalized to  $M_f$  as a function of  $t$  for  $d = 2, 4, 6$ .

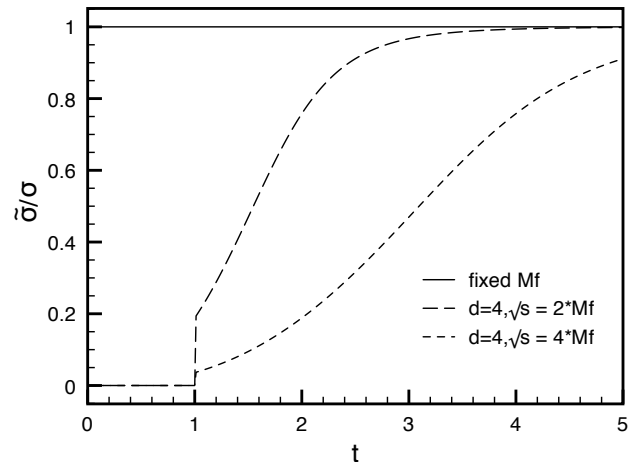


FIG. 3: Dependence of the normalized cross section  $\tilde{\sigma}/\sigma$  on the regularization parameter  $t$  for  $d = 4$  and for  $\sqrt{s} = 2M_f$  ( $\sqrt{s} = 4M_f$ ).

whether the results obtained here remain valid after an improved formulation of the BH threshold, the RG solutions, and the truncation parameter  $t$ . If the results do not change in a more detailed formulation and  $t$  can be determined to be smaller than one no black holes will be produced at future colliders regardless if large extra dimensions exist or not.

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